Evaluate $\int_{-\pi}^{\pi} \frac{2\sin t}{1-\cos t}\,dt$.

SCORE: _____ / 5 PTS

$$= \boxed{\int_{-\pi}^{0} \frac{2\sin t}{1-\cos t}\,dt + \int_{0}^{\pi} \frac{2\sin t}{1-\cos t}\,dt} \quad 1$$

$$\hookrightarrow = \boxed{\lim_{N\to 0^+} \int_{N}^{\pi} \frac{2\sin t}{1-\cos t}\,dt} \quad 1$$

$u = 1-\cos t$

$du = \sin t\,dt$ $\quad \int \frac{2}{u}\,du$

$= 2\ln|u| + C$

$$= \lim_{N\to 0^+} \boxed{2\ln|1-\cos t|}\Big|_{N}^{\pi} \quad 1$$

$$= \boxed{\lim_{N\to 0^+} 2\left(\ln 2 - \ln|1-\cos N|\right)} = \boxed{\infty} \quad \tfrac{1}{2}$$

$$\boxed{\int_{-\pi}^{\pi} \frac{2\sin t}{1-\cos t}\,dt \text{ DIVERGES}} \quad \tfrac{1}{2}$$

1

Evaluate $\int \frac{35-x^2}{x^3+2x^2+5x}\,dx$. $= \boxed{\int\left(\frac{A}{x}+\frac{B(2x+2)+C}{x^2+2x+5}\right)dx}$ 1

SCORE: _____ / 8 PTS

$= \int\left(\boxed{\frac{7}{x}}+\frac{\boxed{-4(2x+2)}\boxed{-6}}{(x+1)^2+4}\right)dx$ 1 1 1

$= 7\ln|x| - 4\ln|x^2+2x+5| - 6\cdot\frac{1}{2}\tan^{-1}\frac{x+1}{2} + C$

$= \boxed{7\ln|x|}\boxed{-4\ln(x^2+2x+5)}\boxed{-3\tan^{-1}\frac{x+1}{2}}+C$ $\frac{1}{2}$ $\frac{1}{2}$ 1

$\boxed{35-x^2 = A(x^2+2x+5)+Bx(2x+2)+Cx}$ 1

$x=0$: $35 = 5A \longrightarrow A=7$

$x=-1$: $34 = 4A - C \longrightarrow C = 4A-34 = -6$

x^2: $-1 = A+2B \longrightarrow B = \frac{1}{2}(-A-1) = -4$

SANITY CHECK $x=2$: $\frac{35-4}{8+8+10} \overset{?}{=} \frac{7}{2} + \frac{(-4)(6)-6}{4+4+5}$

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1 $\boxed{\frac{31}{26} \overset{?}{=} \frac{7}{2} + \frac{-30}{13}} = \frac{91-60}{26} = \frac{31}{26}$ ✓

Evaluate $\int_0^{\infty} \frac{e^{-\frac{1}{y}}}{y^2}\,dy$. **SCORE: _____ / 8 PTS**

$$= \boxed{\int_0^1 \frac{e^{-\frac{1}{y}}}{y^2}\,dy + \int_1^{\infty} \frac{e^{-\frac{1}{y}}}{y^2}\,dy} \; 1$$

$$1 \; = \boxed{\lim_{N\to 0^+} \int_N^1 \frac{e^{-\frac{1}{y}}}{y^2}\,dy} + \boxed{\lim_{M\to\infty} \int_1^M \frac{e^{-\frac{1}{y}}}{y^2}\,dy} \; 1$$

$u = -\frac{1}{y}$

$du = \frac{1}{y^2}\,dy$

$\int e^u\,du = e^u + C$

$$= \lim_{N\to 0^+} \boxed{e^{-\frac{1}{y}}}\Big|_N^1 + \lim_{M\to\infty} e^{-\frac{1}{y}}\Big|_1^M$$

1

$$1 \; = \boxed{\lim_{N\to 0^+} \left(e^{-1} - e^{-\frac{1}{N}}\right)} + \boxed{\lim_{M\to\infty} \left(e^{-\frac{1}{M}} - e^{-1}\right)} \; 1$$

$$= \boxed{e^{-1} - 0 + 1 - e^{-1}} \; 1\tfrac{1}{2}$$

$$= \boxed{1} \; \tfrac{1}{2}$$

Evaluate $\int \frac{3+6x^2-2x^4}{x^3+2x^2+x}\,dx$. $= \boxed{\int \left(-2x+4+\frac{-4x+3}{x(x+1)^2}\right)dx}$ 1

SCORE: _____ / 9 PTS

$$\begin{array}{r} -2x+4 \\ x^3+2x^2+x \enclose{longdiv}{-2x^4 \quad +6x^2 \quad +3} \\ \underline{-2x^4-4x^3-2x^2} \\ 4x^3+8x^2 \\ \underline{4x^3+8x^2+4x} \\ -4x+3 \end{array}$$

$= \boxed{\int \left(-2x+4+\frac{A}{x}+\frac{B}{x+1}+\frac{C}{(x+1)^2}\right)dx}$ 1

$= \int \left(-2x+4 \boxed{+\frac{3}{x}} \boxed{-\frac{3}{x+1}} \boxed{-\frac{7}{(x+1)^2}}\right)dx$ 1 1 1

$= \boxed{-x^2+4x} + \boxed{3\ln|x|} \boxed{-3\ln|x+1|} \boxed{+\frac{7}{x+1}} + C$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$

$\boxed{-4x+3 = A(x+1)^2+Bx(x+1)+Cx}$ 1

$x=0: \quad 3 = A$

$x=-1: \quad 7 = -C \rightarrow C = -7$

$x^2: \quad 0 = A+B \rightarrow B = -A = -3$

SANITY CHECK $x=-2$: $\frac{3+24-32}{-8+8-2} \stackrel{?}{=} 8+\frac{3}{-2}+\frac{-3}{-1}+\frac{-7}{1}$

$\boxed{\frac{-5}{-2} \stackrel{?}{=} 8-\frac{3}{2}+3-7}$ 1

$\frac{5}{2} = 4-\frac{3}{2} = \frac{5}{2}$ ✓

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